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113. Structural Vibration Modes of an Angular Velocity Sensor

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Abstract: The paper deals with finite element modeling of the rotational motion sensor that uses Coriolis effect and vibrating quartz tuning fork to sense angular velocity. The computation model is applicable to both studying the performance of measurement system in various modes of operation and evaluating the indeterminacy of measurement results due to the parameters' deviations and external disturbances.

Keywords: Structural vibrations, sensors, mathematical modeling.

INTRODUCTION

The improvement of sensors providing precision angular velocity measurement as well as the development of new high-performance systems is substantial in a lot of areas of engineering. The increasing complexity of these systems causes the complication of their functioning algorithms and analytical models. Technological advances in the improvement of accuracy and dynamical properties of the systems in question can be expected if the processes taking place in the sensors will be better understood. This implies the use of higher level of abstraction models that deal not only with fundamental principles underlying measurement systems but also with principles of treatment of deviations and uncertainties based on the analysis of instrument structure and the effects of sensitivity to parameters variations and external influences. In many cases the models that are formed on basic physical theory and phenomena of the relevant systems should be completed by more comprehensive consideration.

Methods of development and analysis of such models are time consuming ones and require special knowledge and approaches to mathematical modeling. An example of this attitude could be sensors that include mechanical elements to sense a dynamic variable being measured. When analyzing the behaviour of high-performance systems it is very important for applications to investigate properties of the system in the presence of disturbances of different physical nature, e. g. vibrations. Structural vibration problems present a major hazard and design limitation for interpretation of these systems properties and their effect on system performance. At the sizing stage of a design process models with simplifying assumptions can be applied. But when it comes to a refinement, more accurate techniques, such as models of higher order, modal analysis and optimization, consideration of the distributed dynamics effects become indispensable.

ANGULAR VELOCITY SENSOR AND ITS MODEL. PRINCIPLE OF OPERATION

The topic describes a computational model of the GyroChip family sensor that uses a micromachined quartz element - a vibrating quartz tuning fork - to sense angular velocity. Using the Coriolis effect, the rotational motion about the sensor's longitudinal axis produces a DC voltage proportional to the rate of rotation. The sensor has found a wide spectrum of applications in the automotive, aerospace, defence, industrial, commercial, and medical industries. The description and performance specifications of the sensor are available [1, 2].

The sensor consists of a microminiature double-ended quartz tuning fork and supporting structure, all fabricated chemically from a single wafer of monocrystalline piezoelectric quartz.

The use of piezoelectric quartz material simplifies the active element, resulting in exceptional stability over temperature and time. The drive tines, which constitute the active portion of the sensor, are driven by an oscillator circuit at a precise amplitude that causes the tines to move toward and away from one another at a high frequency (see Fig. 1).

Each tine will have Coriolis force acting on it of:

$$F = 2 \cdot m \cdot \Omega \cdot V_r \tag{1}$$

where m = tine mass, V_r = instantaneous radial velocity, and Ω = input rate

This force is perpendicular to both the input rate and the instantaneous radial velocity. The two drives tines move in opposite directions; the resultant forces are perpendicular to the plane of the fork assembly and in opposite directions as well. This produces a torque that is proportional to the input rotational rate. Because the radial velocity is sinusoidal, the torque produced is also sinusoidal at the same frequency of the drive tines, and in phase with the radial velocity of the tine.

The pickup tines, being the sensing portion of the sensor, respond to the oscillating torque by moving into and out of plane, producing a signal at the pickup amplifier. After amplification, those signals are demodulated into a DC signal proportional to the sensor's rotation. The output signal of the sensor reverses sign with the reversal of the input rate since the oscillating torque produced by the Coriolis effect reverses phase when the direction of rotation reverses.

The sensor will generate a signal only with rotation about the axis of symmetry of the fork. That is the only motion that will, by Coriolis sensing, produce an oscillating torque at the frequency of the drive tines.

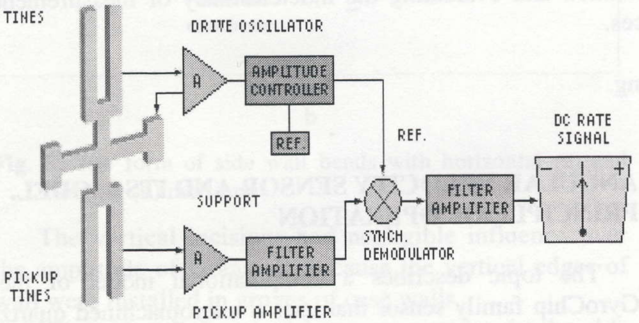


Fig. 1.

LUMPED-PARAMETERS MODEL

The sensor representation given allows definition of organic components of a sensor and describes their qualities and interactive behavior.

Having determined a physical effect and the possibility of a sensing technique, an adequately formulated models enable existing systems to be studied in modes of operation in which they may be called to provide and allow the modeling process to better simulate the system by providing numerical understanding.

The simplest lumped-parameters model of the sensor is presented in Fig. 2.

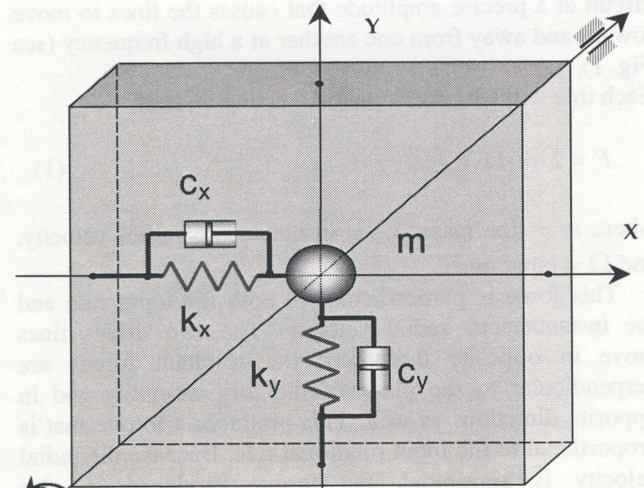


Fig. 2.

It contains a mass able to vibrate in Ox and Oy directions independently. The system is mounted inside of a frame the angular velocity of which is being measured. The coupling between the vibration in the two directions takes place because of the Corriolis acceleration that is directed along Oy as a consequence of the rotation of the frame about Oz and the velocity of the mass along Ox.

The dynamic equation of the system in Fig. 2 can be presented as

$$\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + 2\begin{bmatrix} \vartheta\omega_x & -\Omega \\ \Omega & \vartheta\omega_y \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + 2\begin{bmatrix} \omega_x^2 & 0 \\ 0 & \omega_y^2 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} \hat{f}\cos\omega t \\ 0 \end{Bmatrix}, \quad (2)$$

where $x(t), y(t), \dot{x}, \dot{y}, \ddot{x}, \ddot{y}$ – correspondingly are: displacements, velocities and accelerations of the mass in Ox and Oy directions, $\vartheta = \frac{c}{2\sqrt{km}}$ - damping ratio of vibrations, $\omega_x = \sqrt{\frac{k_x}{m}}$, $\omega_y = \sqrt{\frac{k_y}{m}}$ - natural frequencies of vibrations along Ox and Oy directions, $\hat{f}$ – excitation force amplitude along Ox, ω - excitation frequency, Ω - angular velocity of the frame. The skew-symmetric terms Ω and $-\Omega$ present the gyroscopic part of the matrix and take into account the Coriolis inertia forces. Here we neglect the spin-softening effects caused by the centripetal inertia forces as vibration displacements of the mass are very small. System (2) is linear one and the harmonic response amplitudes can be easily obtained.

The main properties of he system can be commented by using relationships in Fig.3 and Fig. 4.

Fig. 3 presents the amplitude against excitation frequency relationships (AFCH) at different values of the ratio of natural frequencies ω_y / ω_x. By choosing the appropriate value of ω_y / ω_x the AFCH of Oy vibrations has a plateau, which defines the excitation frequency range ensuring the steady level of the response enabling to keep the excitation frequency in the vicinity of resonance.

If for the above mentioned example system having the damping ratio 0.02 we select the natural frequency ratio ω_y / ω_x=1.03, the relationship of response amplitudes (along Oy) and phases against the angular velocity of the rotation of the frame is presented in Fig .4.

In the range of angular velocities $-0.01\omega_x < \Omega < 0.01\omega_x$ nearly linear relationship can be observed. The reverse of the direction of the angular velocity leads to the immediate reverse of the sign of the phase of Oy vibrations.

FINITE ELEMENT MODEL

Investigating the vibrations by means of the finite element model can considerably facilitate the understanding of the operation specifics of the sensor and the quantitative evaluation of the relationship of the output signal against the angular velocity of the outer frame.

The tuning fork is a vibrating piezoelectric plate of a complex geometric shape the side surfaces of which are covered by electrodes enabling to create an electric field inside of the material. In this practical situation the electric field created in the material may be considered as being prescribed, so the piezoelectric phenomena in the plate are governed by the single linear piezoelectricity equation as

$$\{\sigma\} = \left[c^E\right]\{\varepsilon\} - [e]\{E\}; \tag{3}$$

where $\{\sigma\}, \{\varepsilon\}$ - vectors containing the components of elastic stress and strain, $\{E\}$ - vectors containing the components of the electric field, $\left[c^E\right]$ - stiffness tensor under constant electric field, $[e]$ - piezoelectric stress tensor.

As relative displacements of the tuning fork with respect to the rigid rotating frame are being considered, the full acceleration $\{a_F\} = \{a\} + \{a_N\} + \{a_T\} + \{a_C\}$ is being used in the virtual work equation of the finite element as

$$\int_V \delta\{\varepsilon\}^T \{\sigma\} dV + \int_V \rho \delta\{u\}^T \{a_F\} dV = \delta\{U\}^T \{R\}, \tag{4}$$

where symbol δ denotes the virtual quantity, $\{u\} = [N]\{U\}$ - displacement vector of a particle inside the finite element expressed in terms of the form function matrix $[N]$ and the nodal displacement vector $\{U\}$, ρ - density of the material, $\{R\}$ - vector of nodal interaction forces, $\{a\}$ - relative acceleration with respect to the rotating frame; $\{a_N\}$, $\{a_T\}$ normal and tangential accelerations due to the rotation of the frame; $\{a_C\}$ - Coriolis acceleration.

The dynamic equation of the finite element of the tuning fork is obtained as

$$[M]\{\ddot{U}\} + 2\omega[C_1]\{\dot{U}\} + \left([K] - \omega^2[K_1] + \varepsilon[K_2]\right)\{U\} = \{R\} + \{F\} + \omega^2[K_1]\{X\} - \varepsilon[K_2]\{X\};$$

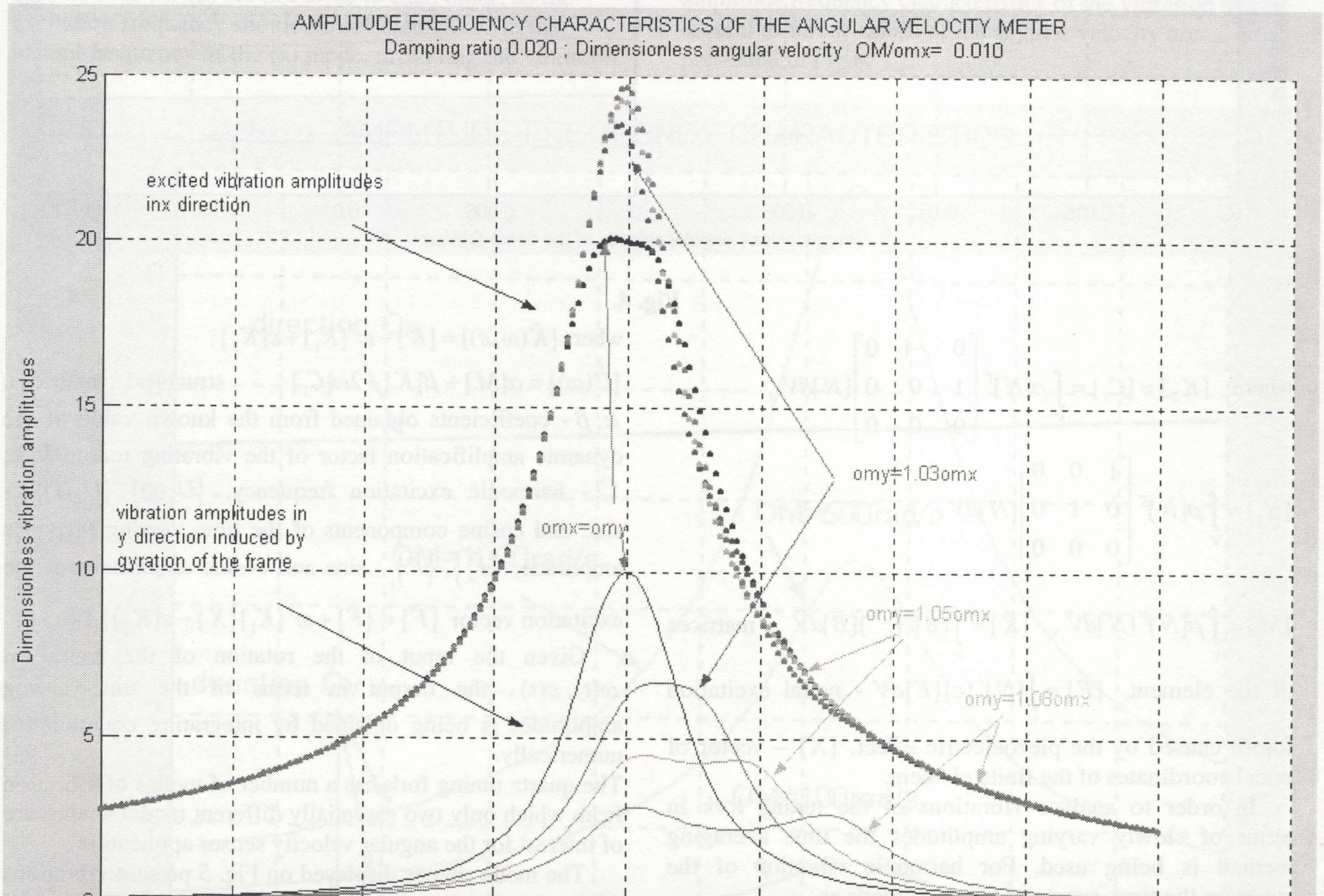


Fig. 3.

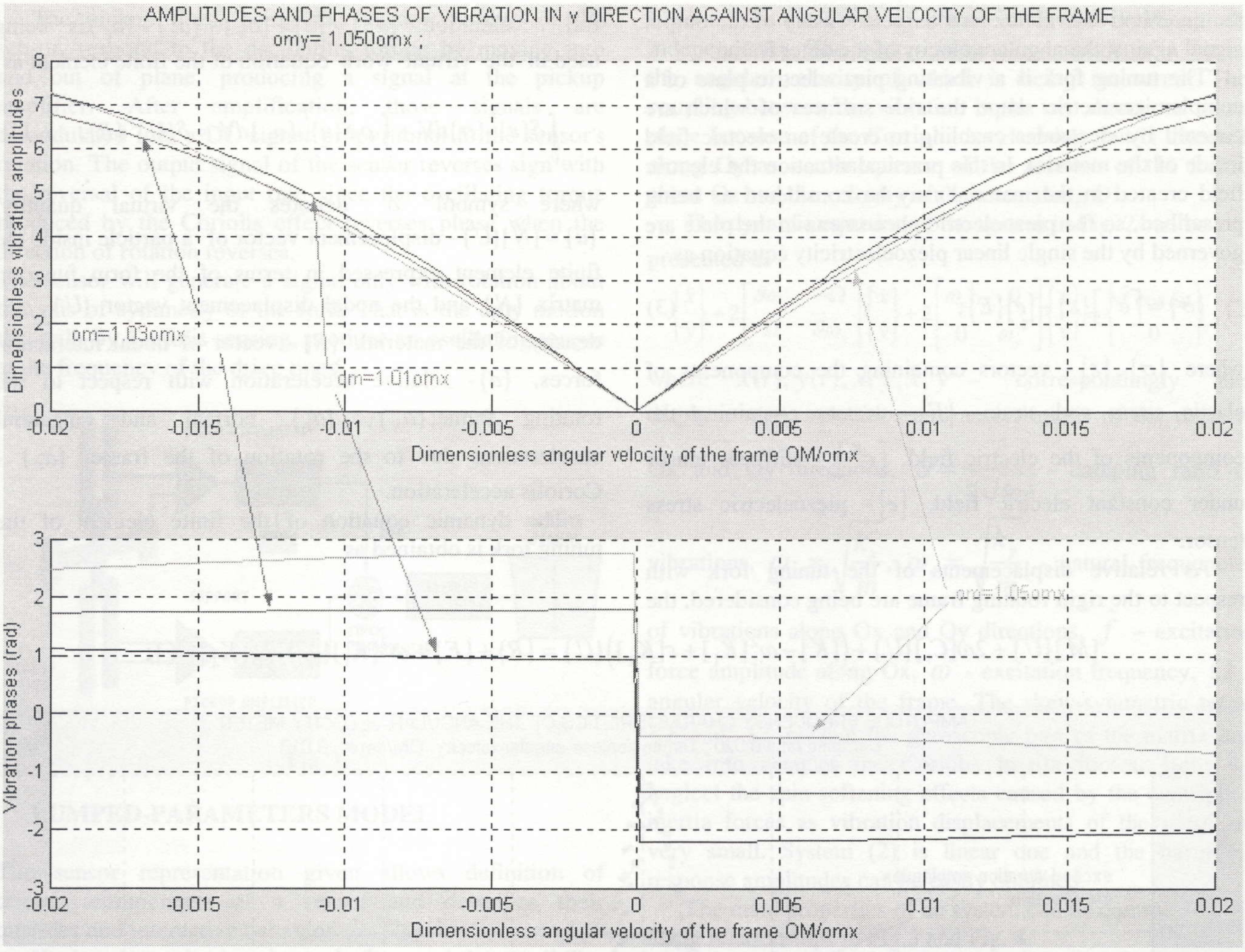


Fig. 4.

where $[K_2]=[C_1]=\int_V \rho[N]^T\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}[N]dV$,

$[K_1]=\int_V \rho[N]^T\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}[N]dV$,

$[M]=\int_V \rho[N]^T[N]dV$, $[K]=\int_V [B]^T[c^E][B]dV$ - matrices of the element, $\{F\}=\int_V [N]^T[e]\{E\}dV$ - nodal excitation forces caused by the piezoelectric effect, {X} – vector of nodal coordinates of the finite element.

In order to analyse vibrations of the tuning fork in terms of slowly varying amplitudes the time averaging method is being used. For harmonic vibration of the structure the time-averaged equation reads as

$$\begin{bmatrix}2\Omega[M] & 0\\ 0 & 2\Omega[M]\end{bmatrix}\begin{Bmatrix}\{\dot{U}_c\}\\ \{\dot{U}_s\}\end{Bmatrix}+\begin{bmatrix}-\Omega[\tilde{C}] & [\tilde{K}]-\Omega^2[M]\\ [\tilde{K}]-\Omega^2[M] & -\Omega[\tilde{C}]\end{bmatrix}\begin{Bmatrix}\{U_c\}\\ \{U_s\}\end{Bmatrix}=\begin{Bmatrix}\{-\tilde{F}_s\}\\ \{\tilde{F}_c\}\end{Bmatrix} \qquad (6)$$

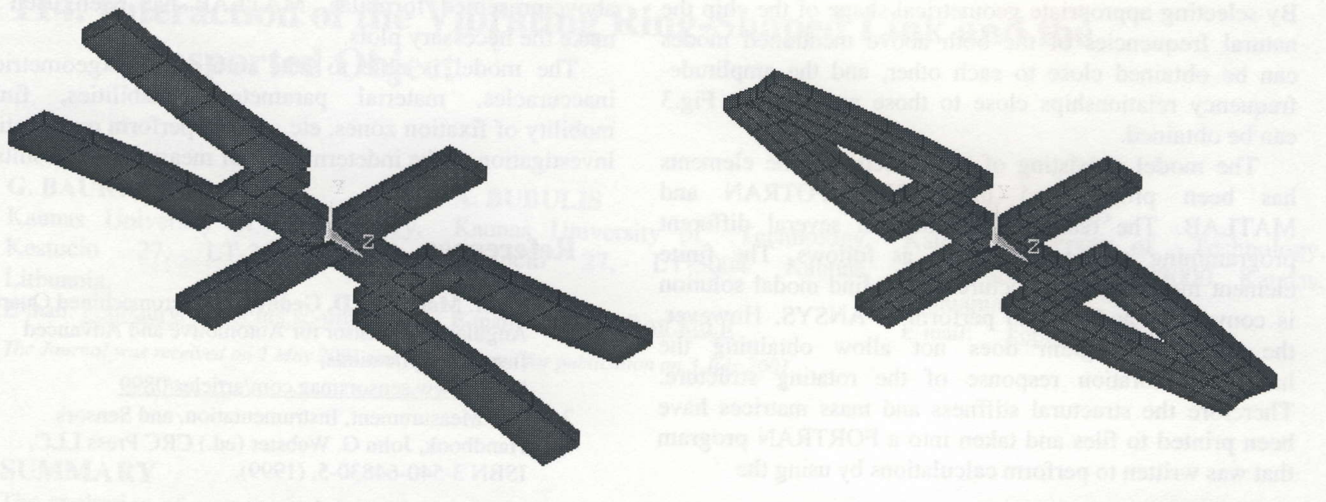
where $[\tilde{K}(\omega,\varepsilon)]=[K]-\omega^2[K_1]+\varepsilon[K_2]$;

$[\tilde{C}(\omega)]=\alpha[M]+\beta[K]+2\omega[C_1]$ - structural matrices, α,β - coefficients obtained from the known value of the dynamic amplification factor of the vibrating tuning fork, Ω - harmonic excitation frequency, $\{U_s(t)\},\{U_c(t)\}$ - sine and cosine components of the time varying response amplitude, $\{\tilde{F}_s\},\{\tilde{F}\}_c$ - sine and cosine amplitudes of the excitation vector $\{\tilde{F}\}=\{F\}+\omega^2[K_1]\{X\}-\varepsilon[K_2]\{X\}$.

Given the input of the rotation of the frame as $\omega(t),\varepsilon(t)$, the output in terms of the time-varying amplitudes is being obtained by integrating equation (6) numerically.

The quartz tuning fork has a number of modes of vibration from which only two essentially different modal shapes are of interest for the angular velocity sensor application.

The mode shapes displayed on Fig. 5 present vibrations of the quartz plate in two perpendicular planes xOz and yOz. If the frame does not rotate, only the (b) vibration mode is being excited by applying the high frequency alternating voltage to plane surfaces of the quartz chip. The vibrations of the (a) mode are obtained if angular velocity is supplied to the frame.



Vibration in Oy direction
Natural frequency 8297Hz

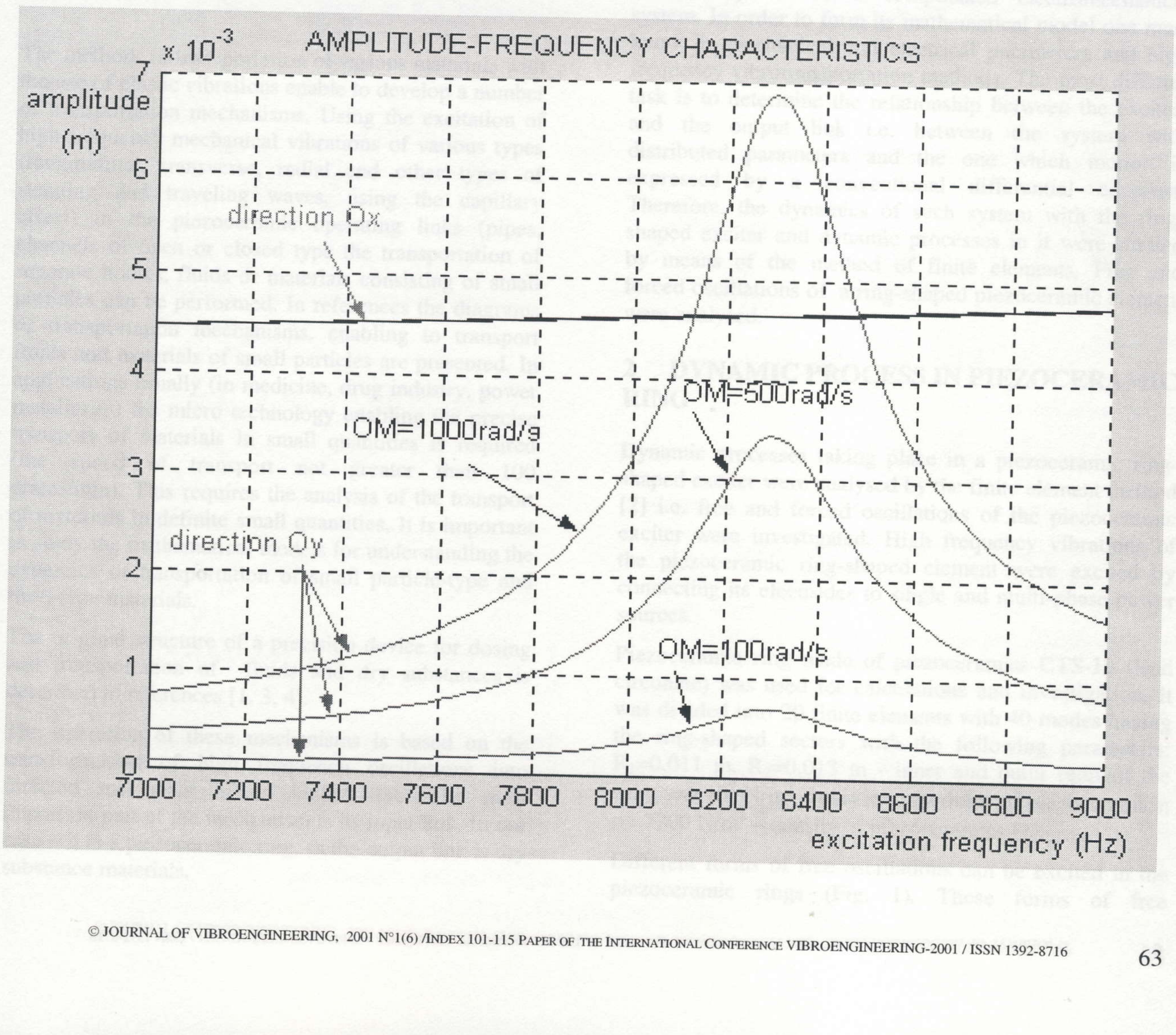
(a)

Vibration in Ox direction
Naturtal frequency 21976Hz

(b)

Fig. 5.

As it can be seen from Fig. 5, the natural frequencies ratio of both modes are not close to each other, so the excitation frequency should be selected closer to the natural frequency of the (a) mode, providing the vibration level proportional to the angular velocity of the frame. The amplitude frequency characteristics of the vibration at several different values of the angular velocity are presented in Fig.6.



By selecting appropriate geometrical shape of the chip the natural frequencies of the both above mentioned modes can be obtained close to each other, and the amplitude-frequency relationships close to those presented in Fig.3 can be obtained.

The model consisting of piezoelectric frame elements has been programmed in ANSYS, FOTRAN and MATLAB. The reason of application several different programming environments was as follows. The finite element model of the structure and to find modal solution is convenient and easy to perform in ANSYS. However, the ANSYS program does not allow obtaining the harmonic vibration response of the rotating structure. Therefore the structural stiffness and mass matrices have been printed to files and taken into a FORTRAN program that was written to perform calculations by using the above-presented formulae. MATLAB has been used to make the necessary plots.

The model is able to take into account geometrical inaccuracies, material parameter instabilities, finite mobility of fixation zones, etc., and to perform quantitative investigation of the indeterminacy of measurement results.

References

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