Finite element computations: modern mathematical insight into physical behavior of structures

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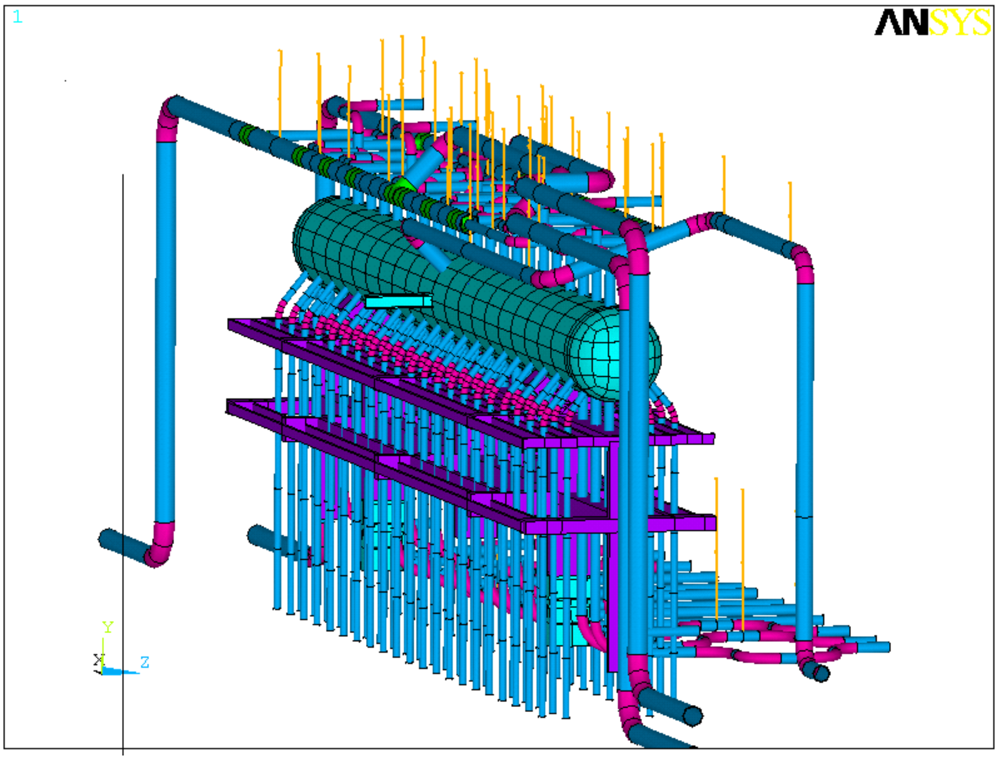
Motivation. From ancient times design and development of engineering structures and systems included the stage of analysis and prediction of their physical behavior. Questions if the future structure has enough strength, what vibrations may occur, what thermal properties it may possess, will it be able to fulfill the expected function or task have always been the most important ones. The simplest and simultaneously most unreliable and dangerous way is to make new designs basing on the past experience only. Nevertheless, a lot of perfect buildings of the past, the genius creators of which had no chance to make proper design calculations, still survive and gratify us. At the same time much more collapsed buildings and bridges, ship catastrophes, automotive crashes and subsequent human deaths occurred because of insufficient design calculations and predictions. The serious and systematic evaluations of designs are available with the advance of mathematics and physics and the theories derived on their basis – mechanics of materials, dynamics, thermal and fluid mechanics, electricity and magnetism. Already at the end of the XIX century the basic principles of the behavior of physical and engineering systems were well understood and described in a strict mathematical form of partial differential equations (PDE). Their solutions enabled a lot of new engineering solutions, which led to explosive advance of engineering and technology. However, along with perfect knowledge of basic principles of physical systems behavior, the possibilities to investigate real objects were still restricted. It was not enough to describe the phenomena by mathematical equations. The useful information is obtained only if the equation is solved. Very often the way of obtaining solutions to equations, which describe real systems are far from being easy and regular. A lot of intellectual minds used their time life long for solving the equations, which represented certain classes of investigated objects. It was hard and slow, often requiring extraordinary abilities and efforts of researchers. The new era in engineering computation came with the advance of computers and the development of the *finite element computational technology*.

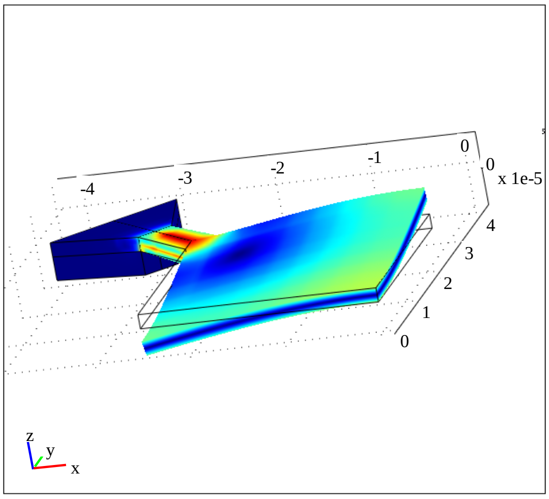
Short history and basic principles. The finite-element method (FEM) originated from the need for solving complex elasticity and structural analysis problems in civil and aeronautical engineering. By late 1950s, the key concepts existed essentially in the form used today. From the mathematical point of view FEM is a numerical method for solving the PDEs, by means of which most physical systems are mathematically described. The basic idea of FEM is as follows. The investigated body or system is presented by a structure of small subdomains (finite elements). Naturally, the overall behavior of the structure, as well as, of a finite element is described by the PDE of the same type. The PDE over an element is approximately replaced by the system of algebraic or ordinary differential equations. The solution obtained over a single element expresses the behavior of the corresponding subdomain of the body to which external actions are applied. The assembly of element equations to overall structural equations describes adequately the physical behavior of the overall body or system. As finite elements may form the structure of any geometrical shape, the bodies or systems of complex geometrical forms are easily represented and their physical behavior simulated. The finite element approach should be treated as a "computational technology" rather than a "method". Computer implementations of FEM invoke a lot of auxiliary numerical functions and subroutines, such as algebraic and ODE solution, eigenvalue analysis, computational geometry and visualization tools, etc.



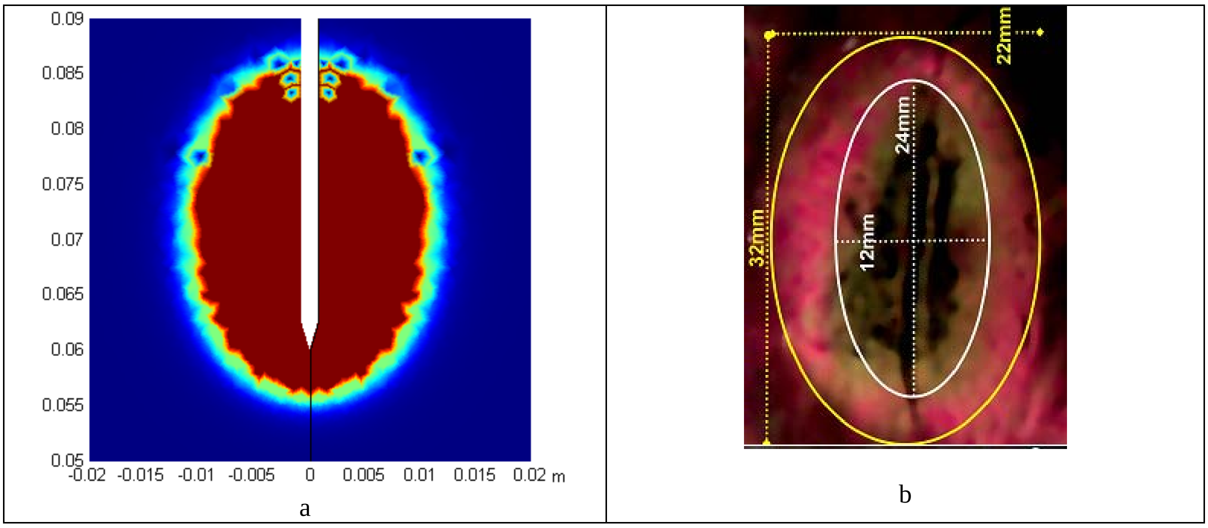
Finite element model for investigation of seismic vibrations of Ignalina NPP separator drum

Research at Kaunas university of technology in the field of FEM and its applications to physical and engineering systems started in the late 1970s at the research centre “Vibrotechnika”. Most new developments were oriented to finite element models of mechatronic systems, the operation principle of which was based on controlled micro-vibrations and nonlinear mechanical interactions of piezoelectric transducers. The next decades were distinguishing by the extra-rapid advance of computers, the transition from mainframes to personal computers and world wide development of “commercial” finite element software systems. The availability of FEM software on the desktop of a researcher enabled to develop new models quickly and efficiently. Simultaneously with the development of application schemes of the method to specific investigation tasks, FEM software systems ANSYS, ALGOR, LS-DYNA, MSC, COMSOL Multiphysics have been studied and applied for creation of complex computational models during numerous local industrial and international projects . An important and extensive research program based on computational modeling of structural behavior has been carried out for structural integrity analysis of Ignalina nuclear power plant (INPP) in order to evaluate and enhance its safety during the residual time of operation. Numerous finite element models of vital units of INPP have been created at Computational technologies centre (earlier Structures reliability centre) of KTU in order to evaluate seismic and other dynamic responses, structural strength and fatigue. The projects oriented to enhancement of personal safety of soldiers (bullet proof clothing), biomedical engineering (radiofrequency ablation of liver tumors), fluid flow and ultrasonic waves phenomena in ultrasonic fluid rate meters, thermal-elastic vibrations of micromechanical devices on chips, etc. required to create complex multifield models, where coupled phenomena of electrical current, thermal conductivity and advection along with mechanical deformation were necessary.

The prospective and necessity of investigations in the field of FEM, as well as, creation of new models in the future is obvious. Though FEM theory is developed and computations carried out for almost sixty years, however, new challenges for creating more efficient and adequate models appear every day. Models of cars, as they behave during the car-crash, the “humanoid” models, which represent the possible injuries of the humans in the car, impact interaction of cars with animals, clothing and wearing comfort models, physical-biomedical models of blood flow and certain treatment procedures – all are still far from being perfect and finished. Computational models developed until certain level of perfection become the objects of commercial value, which can be used and enhanced as new constructions and designs are developed.



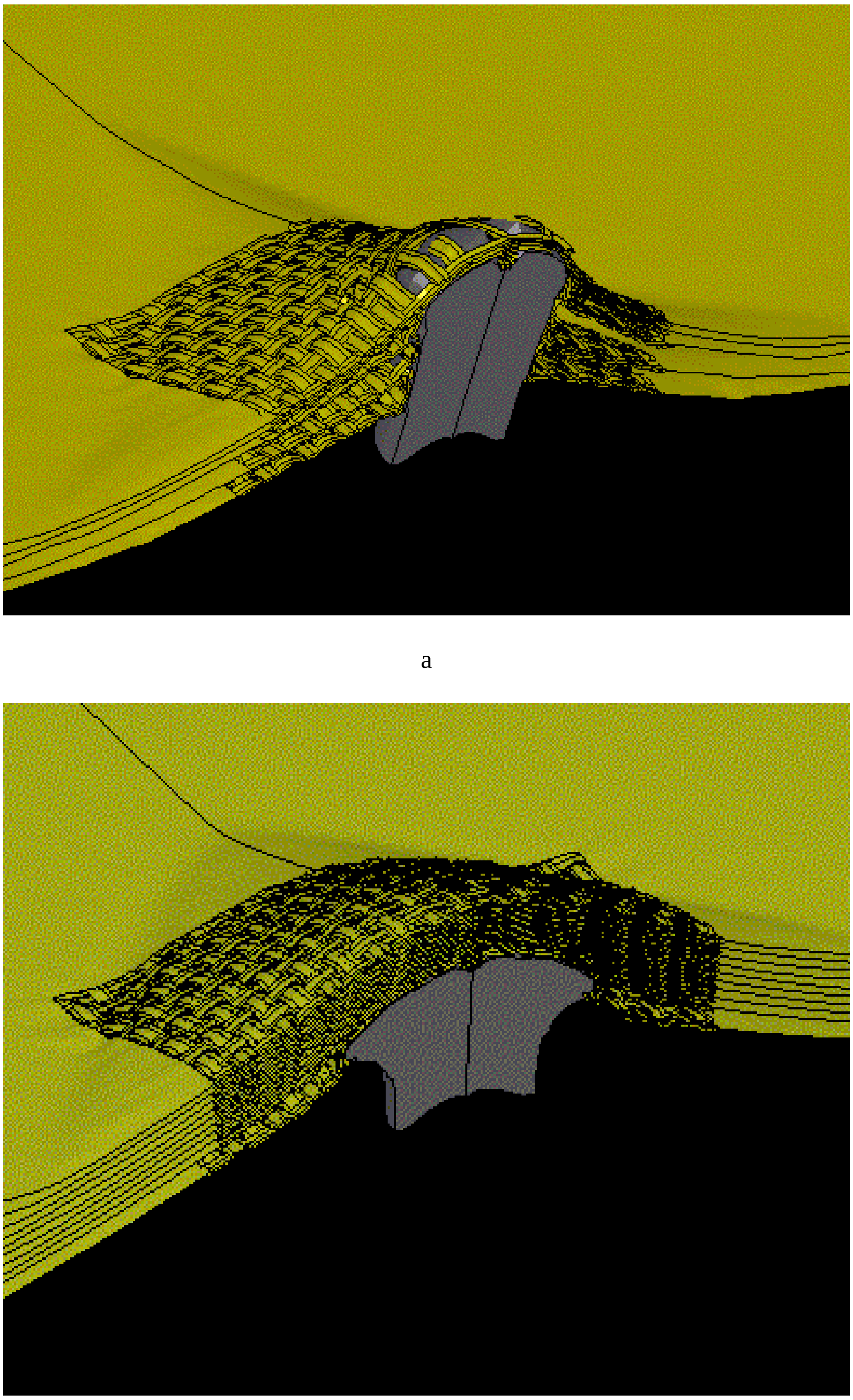
Modelling of thermally-elastic modal vibration of a rectangular MEMS resonator



a

b

Computed and experimental radiofrequency ablation zone of the liver tumor



Modelling of ballistic impact of a bulled against the bullet-proof textile package: penetration through 5 textile layers(a) and bullet hold-up in 10 layers package (b)