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Enhanced Simulation Techniques for Short Wavelength Pulses Propagation through Acoustic-Solid Media

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Enhanced Simulation Techniques for Short Wavelength Pulses Propagation Through Acoustic-Solid Media

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ABSTRACT

The presented approach for reducing the phase and group errors in short wavelength pulses propagation modeling is based upon the modal error minimization. The computational model is built of alike component substructures the matrices of which are obtained by modal synthesis. The necessary modal properties of component substructures are established by solving the total modal error minimization problem for a sample structure the exact modal frequencies of which are known theoretically and where modal frequencies and shapes of a component substructure are used as the design parameters. After the matrices of a component substructure are obtained, they can be used to form structure as higher-order elements. As a result, models having 60-80% of modal frequencies with an error less than 3% can be obtained by using the optimized component substructures. Though the synthesized mass matrices are non-diagonal, the obtained dynamic models are able to simulate short transient waves and wave pulses propagating in elastic or acoustic environments by using only a few nodal points per pulse length. Large computational models are subdivided into sub-domains in which different time and space step sizes are employed. The presented technique enables to enhance considerably the efficiency of the simulation of acoustic-solid problems where great variations of wave propagation velocity in different sub-domains can be observed.

KEYWORDS: modal synthesis; modal error; generalized mass matrices; wave propagation, time sub-stepping

1. INTRODUCTION

The finite element simulation of elastic and/or acoustic waves travelling distances many times exceeding the wavelength is still a challenging problem in computational mechanics. As one of many applications, it is widely used during development of ultrasonic measurement procedures in order to get insight into wave propagation and transformation phenomena taking place inside of the analyzed domain. In such applications the problems of practical value require the product (wave number) × (characteristic dimension of the domain) to be near to100 or even 1000.

The shape of a propagating short wavelength pulse simulated in a discrete mesh is inevitably distorted if the distance traveled by the pulse comprises a large number of lengths of the pulse. As a result, the shape and duration of the simulated pulse become very different from the values expected theoretically. An important source of distortions is the phase error inherently produced by the discrete model. The errors can be and usually are minimized by using very dense meshes, however, this makes the simulation complex and requiring huge computational resources. The modal errors of the computational domain can be regarded as an origin of phase errors. As a consequence of them, different harmonic components of waves comprising the wave pulse propagate with different velocities and produce group errors of wave propagation.

As early as in 1982 different modal frequency convergence features of dynamic models obtained by using lumped and consistent forms of mass matrices have been noticed [2]. The convergence of modal frequencies of dynamic models can be improved by using the 'generalized' form of the mass matrix obtained as a weighted superposition of lumped and consistent mass matrices, [3]. Approaches concentrating on improvement of modal convergence properties and retaining the diagonal form of the mass matrix have been presented as well, [4-7]. The matrices obtained by modal synthesis gave even better results, [8], however, such matrices are non-diagonal. In [8], the minimization problem has been solved for the penalty function representing the modal frequency error of a sample substructure. This work is based upon the approach formulated in [8] where it worked well in 1D case. The results for 2D regular meshes describing elastic and/or acoustic wave propagation have been obtained. As a result, models having 60-80% of modal frequencies with an error less than 3% can be obtained by using the optimized component substructures. Though the synthesized mass matrices are non-diagonal, the obtained dynamic models are able to simulate short transient waves and wave pulses propagating in elastic or acoustic environments by using only a few nodal points per pulse length.

An algorithm for short wavelength pulse simulation presented in [9]. The algorithmic efficiency was based on the explicit strategy of solution and using structures consisting of interacting sub-domains. The mesh roughness of each individual sub-domain could be different and depended upon the value of the wave propagation velocity in it. In this work, the time sub-stepping capability has been added to the program.

2. GENERAL RELATIONS OF MODAL SYNTHESIS

Finite element models of small vibrations and waves in elastic or acoustic continua are presented by the well known semi-discrete structural dynamic equation as

$$[\mathbf{M}]\{\ddot{\mathbf{U}}\}+[\mathbf{K}]\{\mathbf{U}\}=\{\mathbf{R}(t)\}, \qquad (1)$$

where $[\mathbf{M}],[\mathbf{K}]$ - structural mass and stiffness matrices of the element, $\{\mathbf{R}\}$ - nodal vector containing the lumped forces. In problems addressed in this work we assume the damping forces to be very small and omit the damping term.

The structural matrices used in (1) can be expressed by using the modal synthesis relations as

$$[\mathbf{M}]=\left([\mathbf{Y}]^T\right)^{-1}[\mathbf{Y}]^{-1};$$
$$[\mathbf{K}]=\left([\mathbf{Y}]^T\right)^{-1}\left[diag(\omega_1^2,\omega_2^2,...,\omega_n^2)\right][\mathbf{Y}]^{-1}, \qquad (2)$$

where $\omega_1,\omega_2,...,\omega_n$ are the modal frequencies of the model of dimension $n\times n$, and $[\mathbf{Y}]=[\{\mathbf{y}_1\},\{\mathbf{y}_2\},...,\{\mathbf{y}_n\}]$ - the modal shapes. By using relations (2) desirable dynamic properties expressed in terms of known eigenfrequencies and eigenforms can be supplied to model (1).

3. OBTAINING “OPTIMUM” COMPONENT SUBSTRUCTURES

Optimization problem formulation. In wave propagation models large parts of computational domains can be built of alike *component substructures* (CS). As a limit case, a CS may consist of a single element, or may be quite large domain the shape of which is geometrically similar to the shape of the element, Fig.1. Modal properties of a stand-alone CS can be obtained by solving an eigenvalue problem. If desirable, the spectral properties of the CS can be slightly changed by modifying the values of modal frequencies, as well as, the modal shapes. After that its structural matrices can be re-calculated by using relations (2) into which the desirable values of ω_i and $\{\mathbf{y}_i\}$ should be substituted. So we need to *optimally modify the spectral properties of a CS in order to produce the minimum modal frequency error of the whole structure.* The modifications must preserve the physical essence of the unconstrained CS, i.e., the modal frequencies corresponding to rigid body modes have to be zeroes, and the modal shape vectors have to be orthogonal and to express essentially the same shapes as before the modification. Also the total mass of the CS must remain unchanged.

The modal frequency error minimization problem can be formally presented as

$$\min_{\omega_i,\{\mathbf{y}_i\}} \Psi=\sum_{i=r+1}^{\hat{N}}\left(\frac{\hat{\omega}_i-\hat{\omega}_{i0}}{\hat{\omega}_{i0}}\right)^2, \qquad (3)$$

where the penalty-type target function presents the cumulative modal frequency error of the whole structure, $\hat{\omega}_i$ - modal frequency of *i*-th mode of the whole structure, $\hat{\omega}_{i0}$ - its exact value known theoretically or obtained by using a highly refined finite element model, $\omega_i,\{\mathbf{y}_i\},\ i=1,2,...,n$ - modal frequencies and shapes of the individual CS.

To solve (3) for the whole structure presenting a real computational domain is rather expensive and hardly reasonable. A good choice is to perform optimization by assembling CS into *sample domains* of regular shape(linear, rectangular, triangular) the sufficiently large number of exact modal frequencies of which is known. E.g., for linear and rectangular elastic and acoustic domains such modal frequencies are available analytically. In other cases a highly refined model of the sample domain can be used in order to obtain ’nearly exact’ (say, <0.5%) modal frequency values. The size of the sample domain is practically determined by a reasonable amount of calculations. Our numerical experiments demonstrate that often it is enough to performs optimization on the sample domain consisting of only several CSs, and the optimized matrices of a single CS work well if a considerably larger structure is assembled. We didn’t find any theoretical proof of the validity of the approach, however, numerical experiments presented in [1] and in this work illustrate that it works.

Sensitivity functions. Optimization calculations are rather expensive. The target function minimization process can be facilitated by calculating the derivatives of function Ψ with respect to design variables as $\frac{\partial\Psi}{\partial\{\omega_i^2\}}$, $\frac{\partial\Psi}{\partial\{y_{ij}\}}$ $i,j=1,...,n$, where y_{ij} is the displacement of j-th d.o.f of the i-th mode of the CS. The vector containing the full set of the derivatives is the gradient of function Ψ which is used as the search direction.

4. TIME SUB-STEPPING PROCEDURE

In wave-propagation domains consisting of several materials largest time step size is governed by the In our computations we used the central difference time integration scheme with the CFL number $\frac{c\Delta t}{\Delta x}<1$, where c – wave propagation velocity, Δt, Δx - time and space step sizes. Therefore the largest allowable time step size is determined by the domain in which the wave propagation velocity is maximum. In order to reduce redundant computations and increase overall performance of the algorithm the time sub-stepping procedure has been implemented.

We use the structural dynamic equation of a sub-domain as

$$[\mathbf{M}]\{\ddot{\mathbf{U}}\}=\{\mathbf{F}(t)\}, \qquad (4)$$

where $\{\mathbf{F}(t)\}$ - global vector of nodal forces. It contains terms $[\mathbf{C}]\{\dot{\mathbf{U}}\}$ and $[\mathbf{K}]\{\mathbf{U}\}$ from (1) as well as, the external force values.

The central difference time integration scheme is given as follows:

$$v_{i,t+1/2}=v_{i,t-1/2}+\Delta t\cdot a_{i,t}, \qquad (5)$$
$$u_{i,t+1}=u_{i,t}+\Delta t\cdot v_{i,t+1/2}, \qquad (6)$$

where $a_{i,t}$ is acceleration of node *i* at time $\Delta t(n)$, $v_{i,t+1/2}$ – velocity of node *i* at time $\Delta t(n+1/2)$, $u_{i,t+1}$ – displacement of node *i* at time $\Delta t(n+1)$, *h* – time step.

The acceleration of node *i* at time step $\Delta t(n)$, is given by

$$a_{i,t} = \frac{f_{i,t}}{m_i}\,, \qquad (7)$$

where $f_{i,t}$ is the total force at node *i*, and m_i is the mass of the node. A special attention should be paid to the calculation of acceleration of the nodes belonging to the sub-domains boundaries. In uni-dimensional examples presented in [10] the smooth variation of the mass density over the nodes is naturally guaranteed since the meshes have no discontinuities, i.e. all nodes are interconnected. In 2D and 3D cases if individual meshes for separate sub-domains are being used, the nodes on the sub-domains contacting lines called interface lines do not coincide. Therefore, it is not trivial to calculate node's acceleration according to the Eq. (7) since there is no continuity of the internal forces and mass density on the nodes. First, we explain interface nodes processing sequence if both zones had the same time step and were updated at the same frequency, and then we extend this approach to time sub-stepping.

In the following, dummy nodes on the interface lines are being used. These nodes coincide with the fine mesh. This is done on the base of assumption that the internal force contributions from both rough and fine mesh sub-domains on the interface node is more exactly evaluated for the fine mesh. Internal forces from fine mesh boundary nodes are simply copied to the interface nodes. Forces from rough mesh are converted to the equivalent pressure distribution and then cast to interface nodes. In similar way the masses of interface nodes are calculated. Now the displacements of the inner nodes of both sub-domains are updated in accordance with (5), (6) and (7). The displacements of the interface nodes are also found by using the same relations. Then they are back-substituted for the fine mesh boundary nodes, while displacements of the rough mesh nodes are re-calculated using the Lagrange interpolation.

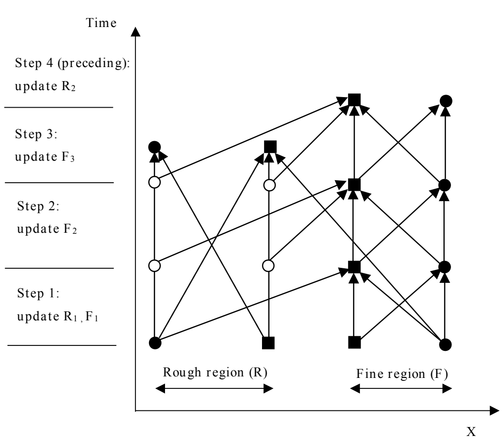

Fig.1 The nodal influence diagram presenting the flow of information of the sub-stepping algorithm.

The time sub-stepping procedure is explained by the nodal influence diagram, Fig.1. Using the same notations as in [10], an arrow represents the influence of a node on the velocity and displacement calculation of another node. A solid dot or square represent the time at which the velocity and displacement of a node are updated, and an open dot represents the time at which the displacement of the node is to be interpolated between two known values. A square is used to mark the interface nodes. The necessity to have duplicated interface nodes (quadrupled in 3D) is determined by the circumstance that fine and rough mesh nodes do not coincide at the interface line, and have to be moved by different time steps. The movement of the rough and fine mesh nodes at different time instances by different amounts inevitably produces solution errors due to the interpolation of displacements in time and space. However, the error usually is in the range of 1% for the integer step ratios up to the value of 3 and about 2% even for non-integer step ratios.

NUMERICAL RESULTS

We begin the modal convergence analysis with the uni-dimensional wave-guide models. Accuracy of modal frequencies of the same uni-dimensional domain obtained by using different models of the same mesh density may differ drastically when using different types of element matrices. Fig.2 presents the modal frequency errors of a uni-dimensional structure assembled of optimized component substructures of different size. The advantage of synthesized component domains in comparison with the lumped $[\mathbf{M}_L]$, consistent $[\mathbf{M}_C]$ and generalized mass matrix $[\mathbf{M}] = k_C[\mathbf{M}_C] + k_L[\mathbf{M}_L]$ is obvious. The generalized mass matrix models are able to produce about 35% error free modal frequencies of the joined domain, meanwhile the models based upon 10-node component domains provides 86% of error free modal frequencies. On the other hand, not all the sizes of component domains can be optimized to give the result of the same quality. E.g., in our investigations we distinguished component domains of dimension 5 and 10 as producing the highest percentage of error free modes.

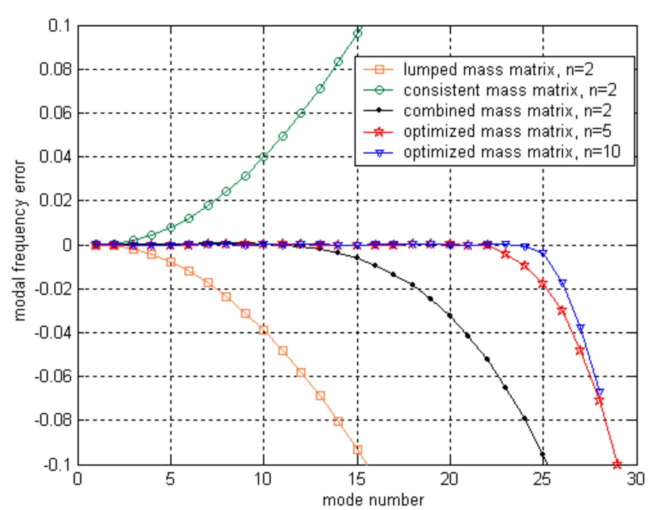

Fig.2. Relative modal errors of a uni-dimensional structure assembled of component substructures by using lumped, consistent, generalized mass matrices and optimized component domains of dimension n=5 and n=10;

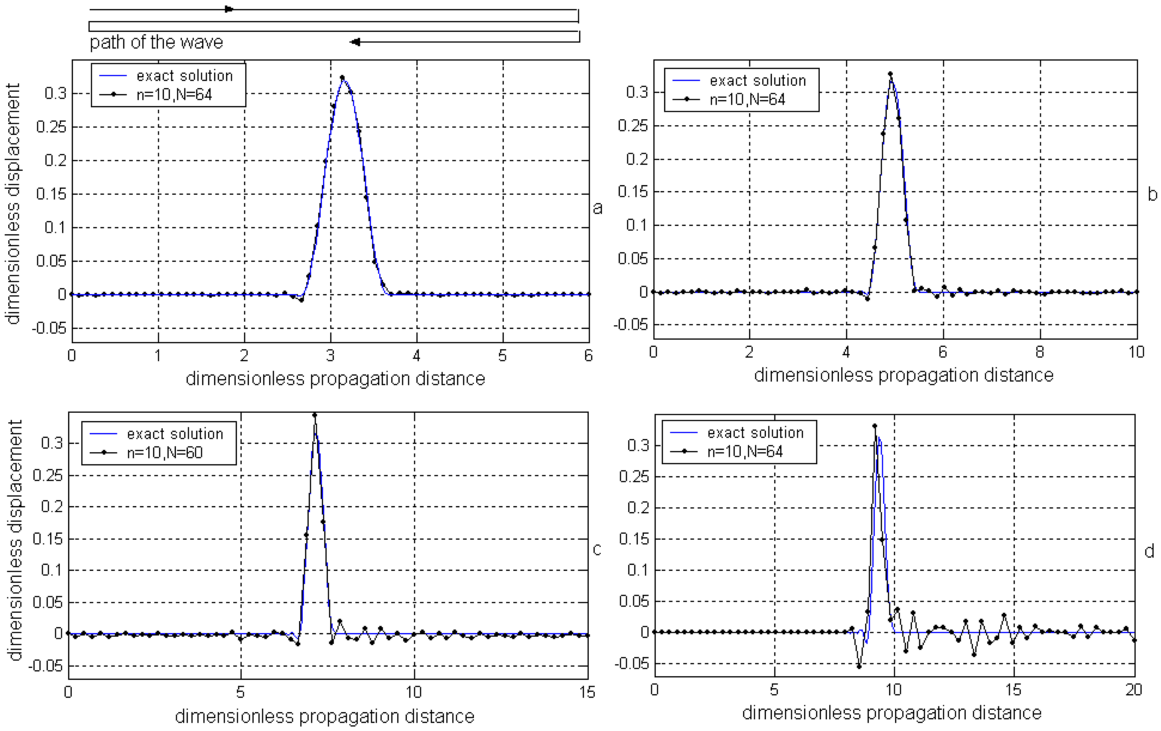

Fig.3. Shape distortion of a propagating wave pulse in the model assembled of seven 10-node component domains.
Number of nodes of the mesh per pulse length:
a- 12 nodes; b – 7 nodes; c- 5 nodes; ; d- 4 nodes;

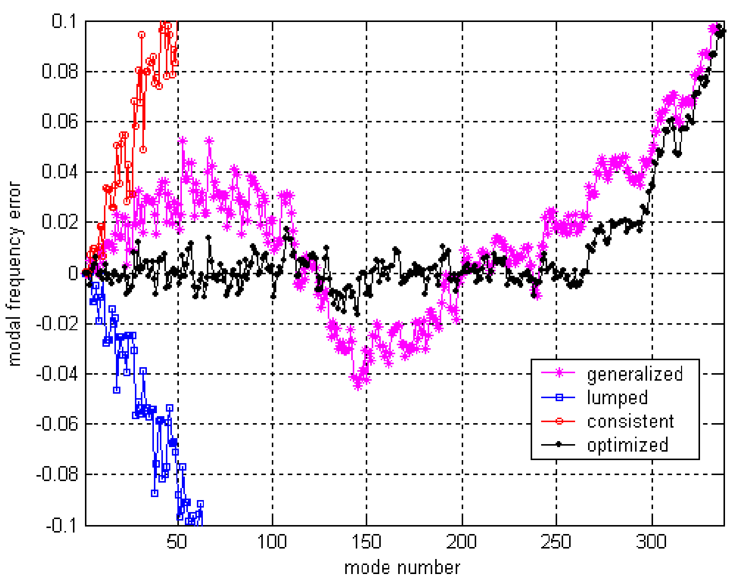

Fig.4.. Relative modal errors of a two-dimensional square domain assembled of component substructures by using lumped, consistent, generalized mass matrices and an optimized component substructure of dimension 5x5

The performance of the 10-node component domain used in the 64 nodes model of the wave-guide simulating the wave pulse propagation is presented in Fig.3. The figure presents the shape distortion of the propagating wave pulse after ~3.5 passages through the joined domain of the wave-guide (see the path of the wave at the top of Fig.3a). 12 or even 7 nodes per pulse length are enough

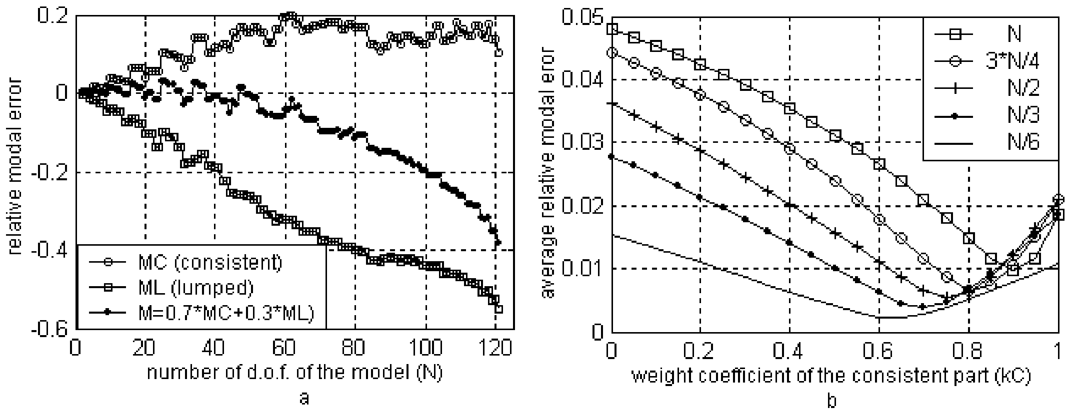

Fig.5. a - modal frequency errors of an acoustic problem in 2D square shaped closed cavity;
b - relationships of average relative errors of modal frequencies against the weight coefficient of the consistent component of the generalized mass matrix.

for simulating the pulse propagation over quite a large distance, Fig.3,a,b. The model actually works satisfactorily also at very rough meshes of 5 or 4 points per pulse length, Fig3,c,d. At the same conditions, the conventional lumped or consistent mass matrix models produce the numerical noise larger then the signal itself and no resemblance of the pulse shape would be seen in the picture.

As a two-dimensional example we present the relative modal frequency error relationships for the elastic wave propagation problem formulated in a square shaped domain. The basic properties of models described by using different forms of mass matrices are briefly explained in Fig.4. Relative modal frequency errors of the square domain obtained by using the consistent, lumped and generalized mass matrices are presented. Qualitatively, the general character of the curves is very close to the results obtained for a uni-dimensional domain presented in Fig.2. The optimized form of the mass matrix gives the minimum cumulative value of the relative modal frequency error.

As acoustic wave propagation model we present the modal frequency error relationships for the acoustic problem formulated in a square shaped closed cavity. The exact modal frequencies can be expressed as $\omega_{(m,n)0} = \pi\sqrt{\frac{E}{\rho}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$, where *a,b* – lengths of the sides of the rectangular. Here the square domain is being analysed, *a=b*. The basic properties of models described by using different forms of mass matrices are briefly explained in Fig.5. Relative modal frequency errors of the square domain obtained by using the consistent, lumped and generalized mass matrices are presented in Fig.5,a. Evidently, there exists an optimum weighted combination of the generalized and lumped matrices $[\mathbf{M}] = k_C[\mathbf{M}_C] + k_L[\mathbf{M}_L]$. The reasoning for the choice of value k_C can be understood from Fig5,b, where the relationships of average modal frequency error taken as

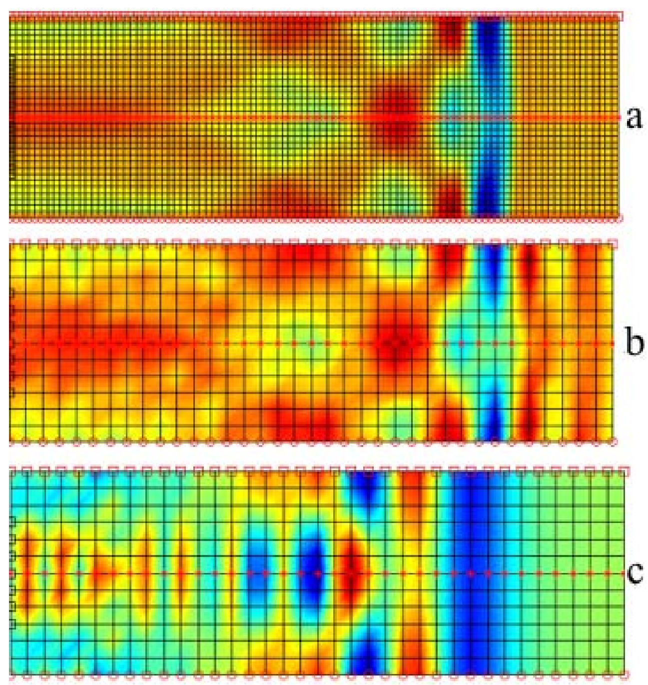

Fig.6. Contours of the dimensionless acoustic pressure field caused by the propagating wave pulse at a certain moment of time:

a-obtained in a fine mesh, can be regarded as “near-to-precise” solution;

b-obtained in a roughly-meshed structure by assembling optimized 5x5 component substructures;

c-obtained in the same roughly-meshed structure as in (b) by using lumped mass matrices. Solution has little resemblance with the exact one.

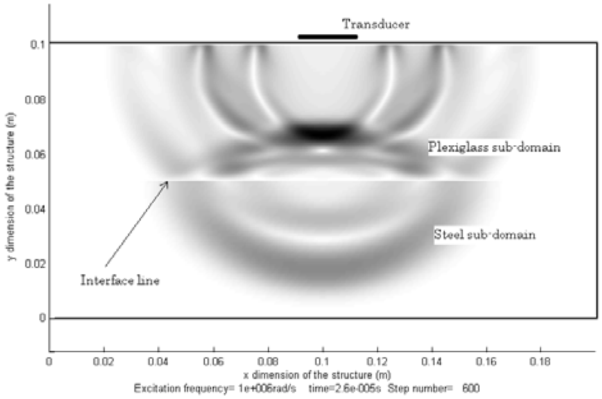

Fig7. Displacement contour plot obtained by using the time substepping technique. Steel domain: mesh size 1.00 mm, time step 8.50e-8 s; Plexiglass domain: mesh size 0.54 mm, time step 4.25e-8 s.

square root of sum of squares $\frac{1}{N}\sqrt{\sum_{i=1}^{N}\left(\frac{\omega_i-\omega_{i0}}{\omega_{i0}}\right)^2}$ against the value k_C are presented. Each curve describes the cumulative error values obtained by taking sums over a different number of modes: N (summation over all modes), 3*N/4, N/2, etc. As it is impossible to get very small error values over all modal frequency range, the optimum values of k_C are slightly different in each case. Practically, for minimum numerically caused distortion of propagating wave pulses a reasonable choice is $k_C = 0.7,\ k_L = 1 - k_C = 0.3$.

As a result the optimized matrices allow to perform the wave propagation simulations by using only few nodes per wavelength. For the sake of comparison the contours of acoustic pressure distribution caused by a propagating wave pulse at a certain moment of time is presented in Fig.6. Obviously, the optimised matrices allow much rougher meshes to present the wave propagation with the desirable accuracy.

5. CONCLUSIONS

The mass and stiffness matrices of component substructures have been obtained such that after assembling them a larger model the convergence of modal frequencies is as high as possible. The method is based upon the minimization of the modal frequency errors of some selected sample domains and then using the obtained component domain matrices for assembling the real computational domains. The obtained mass matrices are non-diagonal. Once calculated, the component substructure matrices can be used to form any structure and may be interpreted as higher-order elements. The latter result has not been proved theoretically, however, illustrated by numerical examples.

When compared with lumped, consistent or generalized mass matrices, the matrices obtained by modal synthesis and optimization produce significantly better results. The models able to present very close-to-exact (less than 0.5% error) modal frequency values of more than ~80% of the total modal frequency number can be obtained. Though the method is illustrated basically by means of uni-dimensional examples, it is formulated for 2D and 3D domains as well.

The obtained dynamic models can be used primarily for modelling short transient waves and wave pulses propagating in elastic or acoustic environments. The distinguishing feature of such models is their ability to present the wave pulse by using only few nodal points per wavelength.

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