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PARAMETERS IDENTIFICATION OF VIBRATION MODELS OF HUMANS DURING STILL STANDING

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Abstract: The article analyses a model of single degree of freedom inverted pendulum controlled by proportional, integral and differential (PID) controller which is used to simulate the vibration of pressure center (COP) of the human body in forward-backward direction during still standing. Numerical experiments revealed that certain traditionally used approaches of the disturbance torque generation should be reconsidered when parameters of a particular human are being identified. The article presents a new disturbance torque generation approach based on the components frequency analysis of the center mass vibration (COM)

Keywords: controlled inverted pendulum, human posture, still standing, optimal control, sensitivity functions, disturbance vibrations.

1. Introduction

The time law of small vibrations of the centre of pressure (COP signal) of humans in standing position (human posture) is one of the most popular ways to measure standing stability of a person, see [1, 5, 6]. It is referred to as a stabilogram. COP signal is a collective outcome of various systems that maintain body upright.

Barauskas and Krušinskienė [3] used inverted pendulum with proportional, integral and differential (PID) controller and identified stabilogram parameters of the particular human. The employed method was based on sensitivities of the penalty-type error function to small variations of model parameters. Authors noted that it would be purposive to investigate accuracy of the method by using other types of the penalty-type functions. Krušinskienė [7] proposed to employ stabilogram diffusion function (SDF) for this purpose.

In this work a numerical experiment which employs SDF for calculating penalty-type function is presented. The obtained results are compared to [3]. The analysis of the results revealed that in both cases the identification of parameters could be much more precise if other approaches of model's disturbance torque generation would be considered.

This work presents a new disturbance torque generation approach based on the frequency components analysis of the centre of mass (COM) recorded during physical experiments.

2. Methods

2.1 Controlled inverted pendulum model

In this work the vibration of COP of a human body in forward-backward direction during still standing is generated using an inverted pendulum model with a single degree of freedom (DOF) controlled by PID controller:

$$I\ddot{u}(t) - mghu(t) = w(u, \dot{u}, \{p\}) \qquad (1)$$

where: m - body mass; I – mass moment of inertia of the body about the ankle joint; h – distance of the centre of mass (COM) from the ankle joint; u - sway angle; g – gravitational acceleration; w – non-linear force vector:

$$w(u, \dot{u}, \{p\}) = T_d(t) - T_c(t);$$

T_d - cumulative disturbance torque obtained from the equation presented in [8]:

$$T_d(t) + B\dot{T}_d(t) = Ax(t);$$

$x(t)$ - random number; A, B - low-filter coefficients; T_c - corrective torque implemented as a PID controller:

$$T_c(t) = K_P u(t) + K_I \int_t u(t)dt + K_D \dot{u}(t);$$

$\{p\} = \{K_P, K_I, K_D\}$ – parameters of the controlled inverted pendulum (CIP) model.

The position of COP, which is measured during experiments, is calculated from u according to Peterka [8]:

$$u_{COP}(t) = hu(t) - \frac{I\ddot{u}(t)}{mg} = au(t) - b\ddot{u}(t)$$
$$a = h; b = \frac{I}{mg} \tag{2}$$

2.2 Stabilogram diffusion function and error function formulation

Stabilogram diffusion function was introduced by Collins and De Luca [5] and is an average estimate of difference of two COP positions separated by the time Δt :

$$u_{SDF}(\Delta t) = \frac{\int_0^{T-\Delta t} \left(u_{COP}(t+\Delta t) - u_{COP}(t)\right)^2 dt}{T - \Delta t} \tag{3}$$

where: $u_{COP}(t)$ - COP coordinate at time moment t, u_{SDF} - SDF, T - time of experiment.

According to CIP parameters identification algorithm presented by Barauskas and Krušinskienė [3], error function J is the difference of the model SDF and SDF of the physical experiment:

$$J(\Delta t) = \int_0^{\tau} \frac{1}{2}\left(u_{SDF}(\Delta t) - u_{REF\_SDF}(\Delta t)\right)^2 d(\Delta t) \tag{4}$$

where: u_{SDF} – SDF of COP signal generated by CIP model; u_{ref_SDF} – reference COP signal SDF (computed from COM signal recorded during physical experiment u_{ref}); τ - maximum value of Δt. Inserting (2) and (3) would result that error function J reads as:

$$J(\Delta t) = \int_0^{\tau} \psi(u, \Delta t) d(\Delta t) \tag{5}$$

where:

$$\psi(u, \Delta t) = \frac{1}{2}\left(\frac{\int_0^{T-\Delta t} \left(au(t+\Delta t) - b\ddot{u}(t+\Delta t) - au(t) + b\ddot{u}(t)\right)^2 dt}{T - \Delta t} - u_{REF\_SDF}(\Delta t)\right)^2 \tag{6}$$

2.3 Error function minimisation

Error function minimization is based on the sensitivity functions method which was thoroughly described by Barauskas and Ostasevičius [2]. The application of the method to CIP model parameters identification was presented by Barauskas and Krušinskienė [3]. The minimum condition of the error function reads as

$$\frac{\partial J}{\partial p} = 0 .$$

Conjugate variables are used in order to express the variation of the error function in terms of $\{p\}$. The basic variation relation reads as:

$$\delta J = \frac{\partial J}{\partial p} \delta p$$

where:

$$\frac{\partial J}{\partial p} = \int_0^T \left((\lambda + \dot{\mu} + \ddot{\eta}) \frac{\partial w(z_T, \dot{z}_T, p)}{\partial p} \right) dt ; \tag{7}$$
$$z(t) = u(t + \Delta t) - u(t) .$$

Time laws of conjugate variables $\lambda(t), \mu(t), \eta(t)$ are obtained by time integration of the conjugate differential equations as

$$\begin{cases} \ddot{\lambda} m - \dot{\lambda}\tilde{c} + \lambda(\tilde{k} - \dot{\tilde{c}}) - \mu\dot{\tilde{k}} - \dot{\eta}\dot{\tilde{k}} = \dfrac{\partial \psi}{\partial u}; \\ \ddot{\mu} m - \dot{\mu}\tilde{c} + \mu\tilde{k} - \dot{\eta}\dot{\tilde{c}} + \eta\dot{\tilde{k}} = -\dfrac{\partial \psi}{\partial \dot{u}}; \\ \ddot{\eta} m - \dot{\eta}\tilde{c} + \eta\tilde{k} = \dfrac{\partial \psi}{\partial \ddot{u}} \end{cases} \tag{8}$$

with the following boundary conditions:

$$\lambda_T = \dot{\lambda}_T = \dot{\mu}_T = \dot{\eta}_T = 0;$$
$$\mu_T = \frac{\partial \varphi}{\partial u_T}, \eta_T = -\tilde{k}_T^{-1} \frac{\partial \varphi}{\partial \dot{u}_T} ,$$

where

$$\tilde{c} = c - \frac{\partial w(u, \dot{u}, p)}{\partial \dot{u}};$$
$$\tilde{k} = k - \frac{\partial w(u, \dot{u}, p)}{\partial u}.$$

2.4 Error function minimization by employing the stabilogram diffusion function

The sensitivity functions method applied to CIP parameters identification employing SDF was presented by Krušinskienė [7]. Differential equation of CIP model (1) is considered together with the error function (5) used as a target function. The conjugate variables are found from (8) together with (6):

$$\begin{cases} \ldots - \mu k - \dot{\eta} \dot{k} = 2a \int_0^{T} \left(az(t) - b\ddot{z}(t)\right) dt \left(\frac{\int_0 \left(az(t) - b\ddot{z}(t)\right)^2 dt}{T - \Delta t} - u_{REF\_SDF}(\Delta t) \right); \\ \dot{\mu} m - \dot{\mu} \bar{c} + \mu \bar{k} - \dot{\eta} \dot{\bar{c}} + \eta \dot{\bar{k}} = 0; \\ \dot{\eta} m - \dot{\eta} \bar{c} + \eta \bar{k} = 2b \int_0^{T-\Delta t} \left(az(t) - b\ddot{z}(t)\right) dt \left(\frac{\int_0^{T-\Delta t} \left(az(t) - b\ddot{z}(t)\right)^2 dt}{T - \Delta t} - u_{REF\_SDF}(\Delta t) \right) \end{cases}$$

With initial conditions:

$$\lambda_T = \dot{\lambda}_T = \mu_T = \dot{\mu}_T = \eta_T = \dot{\eta}_T = 0$$

The derivative $\frac{\partial J}{\partial p}$ is found from (7):

$$\frac{\partial J}{\partial p} = \begin{Bmatrix} \int_0^T \left(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t)) z(t)\right) dt; \\ \int_0^T \left(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t)) \int_0^t z(\tau) d\tau \right) dt; \\ \int_0^T \left(-(\lambda(t) + \dot{\mu}(t) + \ddot{\eta}(t)) \dot{z}(t)\right) dt \end{Bmatrix} \quad (9)$$

The Steepest Descent method was used to minimize the target function J and solve equation (9).

3. Implementation

The CIP model parameters identification algorithm was implemented in Matlab7. Disturbance torque T_d time signal was produced by Matlab function "randn". COP signal was recorded during physical experiments by using sample rate of 100 Hz during 60 s. The COM reference signal u_{ref} was calculated from the COP passed through 4th order low-pass Butterworth filter [4]. Body mass and height of COM are measured data of an individual person. We used the mass moment of inertia I = 76 kg·m^2, mass m=60 kg and distance of COM from the ankle h = 1,13m. The initial model parameters set $\{p\}$ was chosen from Peterka [8]:

K_P=1470 N·m·rad^{-1},
K_I=14 N·m·rad^{-1}·s^{-1},
K_D=200 N·m·rad^{-1}·s.

The initial set $\{p\}$ is known as being able to produce the realistic COP signal [8].

4. Results

The experiments were conducted in order to find the best coincidence of the experimental COM signal (Fig. 1, signal Experimental U_{COM}) with the COM signal produced by CIP model (Fig. 1, signals Identified U_{COM} and Before identification U_{COM}).
During identification procedure the model parameters were identified as follows:

K_P = 1472.15 N·m·rad^{-1},
K_I = 15.73 N·m·rad^{-1}·s^{-1},
K_D = 231.01 N·m·rad^{-1}·s.

SDFs calculated from COM signal recorded during physical experiment and CIP model before and after parameters identification are presented in Fig. 2. It may be noted that the CIP model SDF (Fig. 2, signal SDF before identification) was shifted (Fig. 2, signal Identified SDF) towards SDF from the physical experiment (Fig. 2, signal Experimental SDF). However the COM signal calculated from CIP model with identified parameters (Fig. 1, signal Identified U_{COM}) could not be treated as having good coincidence with experimental U_{COM} (Fig. 1, signal Experimental U_{COM}).
It can be observed that both COM signals produced by CIP model are closer to the disturbance torque T_d (Fig. 3) rather than to the experimental COM signal.
Several different disturbance torque T_d instances were generated and identification experiments were conducted. All of them confirmed the confidence that the major factor which influences the COM signal produced by CIP model is the disturbance torque rather than minor changes in controlled parameters set $\{p\}$.
The obtained results were compared to the results presented by Barauskas and Krušinskienė [3] and Peterka [8]. The analysis revealed that the CIP parameters identification method employing SDF was able to identify realistic parameters.

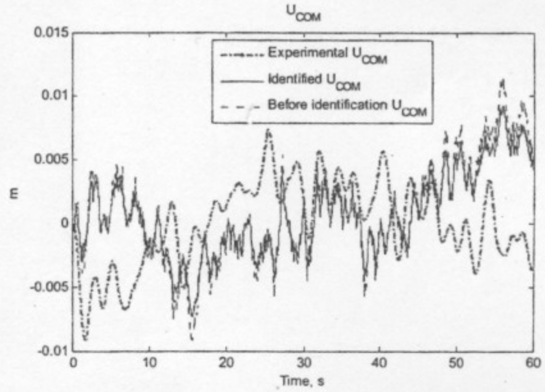

Fig. 1. Comparison of COM signal produced during physical experiment and by CIP model before and after parameters identification

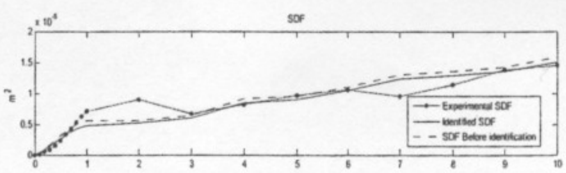

Fig. 2. Comparison of SDF produced by COM signal recorded during physical experiment and by COM signals calculated from CIP model before and after parameters identification

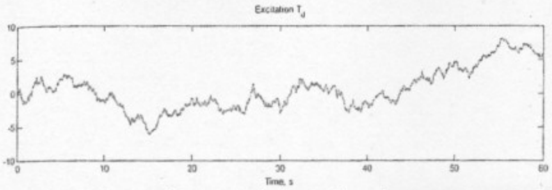

Fig. 3. CIP model cumulative disturbance torque T_d produced as a low-filtered white noise

5. Alternative method for the generation of the disturbance torque

Both techniques presented by Barauskas and Krušinskienė [3] and in this work were able to identify CIP model parameters, which may be estimated as realistic when compared to results presented by Peterka [8]. Nevertheless it is obvious that more accurate data could be provided if more adequate time law of the disturbance torque T_d would be generated. This should be done in a way, which is different from the one proposed by Peterka [8] when the parameters of a particular human being are identified.

The idea is to find an approximation of the disturbance torque which would be dependent on the COM signal recorded during physical experiment.

5.1. Disturbance torque generation from COM signal frequency components

Let us rewrite the disturbance torque T_d and COM signals in Fourier series form:

$$T_d(t)=\frac{1}{2}a_0+\sum_{n=1}^{N}\left[a_n\cos(nt)+b_n\sin(nt)\right], \quad (10)$$

$$a_n=\frac{1}{T/2}\sum_{0}^{T}T_d(t)\cos(nt),$$

$$b_n=\frac{1}{T/2}\sum_{0}^{T}T_d(t)\sin(nt),$$

$$u(t)=\frac{1}{2}q_0+\sum_{n=1}^{N}\left[q_n\cos(nt)+p_n\sin(nt)\right], \quad (11)$$

$$q_n=\frac{1}{T/2}\sum_{0}^{T}u(t)\cos(nt),$$

$$p_n=\frac{1}{T/2}\sum_{0}^{T}u(t)\sin(nt),$$

where: n - a specific frequency harmonic; N – is a total number of harmonics that the signal consists of.

After rearranging and differentiation of the differential equation of CIP model (1) reads as follows:

$$I\dddot{u}(t)+K_D\ddot{u}(t)+(K_P-mgh)\dot{u}(t)+K_I u(t)=\dot{T}_d(t) \quad (12)$$

After substitution of (10) and (11) into (12) CIP model equation reads as follows:

$$\begin{aligned}&I\left(\sum_{n=1}^{N}\left[q_n n^3\sin(nt)-p_n n^3\cos(nt)\right]\right)+\\&+K_D\left(-\sum_{n=1}^{N}\left[q_n n^2\cos(nt)+p_n n^2\sin(nt)\right]\right)+\\&+(K_P-mgh)\left(\sum_{n=1}^{N}\left[-q_n n\sin(nt)+p_n n\cos(nt)\right]\right)+\\&+K_I\left(\frac{1}{2}q_0+\sum_{n=1}^{N}\left[q_n\cos(nt)+p_n\sin(nt)\right]\right)=\\&=\sum_{n=1}^{N}\left[-a_n n\sin(nt)+b_n n\cos(nt)\right]\end{aligned} \quad (13)$$

After rewriting equation (13) for each frequency harmonic $n=1...N$ and assuming that $q_0=0$, the set of equations with 2N size will follow as

$$\begin{cases}-Iq_n n^2+K_D p_n n+(K_P-mgh)q_n-\dfrac{K_I p_n}{n}=a_n^A\\-Ip_n n^2-K_D q_n n+(K_P-mgh)p_n+\dfrac{K_I q_n}{n}=b_n^A\end{cases} \quad (14)$$

From equations (14) Fourier coefficients of model's cumulative disturbance torque T_d are calculated and the T_d signal is reconstructed in the time domain. Such disturbance torque signal generation method could be used for CIP parameters identification for a particular human being.

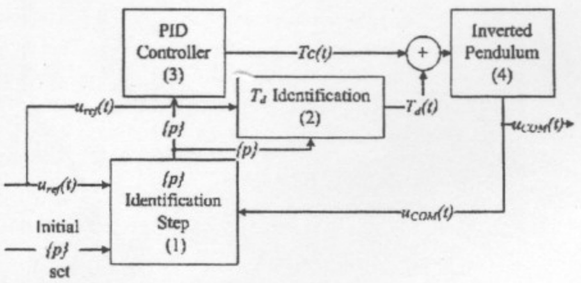

Fig. 4. Iterative CIP model's parameters identification procedure when the disturbance of the model is not known in advance

5.2. General case of parameters identification of CIP model

The disturbance torque T_d generation algorithm presented in Chapter 5.1 is valid only with a fixed set of CIP model parameters *{p}*. It should be noticed that during the parameters identification procedure presented in Chapter 2 only COP signal measured during physical experiment is known. The parameters set *{p}*, as well as, the disturbance torque T_d are not known in advance. In this case the iterative procedure presented in Fig. 4 must be performed.

The procedure consists of 4 major blocks explained in the flow chart, Fig. 4. The first block is "*{p}* Identification Step" which is an iteration of the algorithm presented in Chapter 2. The result of it is the determined parameters set *{p}*, which is the input of block 2. Block 2 contains the algorithm presented

in Chapter 5.1. It determines the disturbance torque T_d of the CIP model at the fixed set of parameters *{p}*. The parameters set *{p}* calculated in block 1 along with the disturbance torque T_d calculated in block 2 are inputs of blocks 3 and 4 which together produce the CIP model as described by equation 1. The result of the algorithm is the time law of vibration of COM $u_{COM}(t)$, which is sent back to block 1 in order to evaluate the coincidence of model's COM coordinate against the experimental signal of COM (u_{ref} signal in Fig. 4).
The same algorithm may be applied to more complex models with several DOF-s.

6. Conclusions

The parameters identification method employing stabilogram diffusion function demonstrated the ability to identify realistic CIP model parameters. Nevertheless the experiments revealed that COM signal produced by CIP model is highly dependent on the cumulative disturbance torque T_d of the model.
A new cumulative disturbance torque generation approach based on experimental COM signal frequency components analysis is being introduced in this paper. The introduced method is valid only with a fixed set of CIP model parameters *{p}*. The identification of the cumulative disturbance torque T_d and the set of parameters *{p}* is performed by iterative procedure.
Very probably such an approach could provide more accurate results of CIP parameters identification. The algorithm may be easily extended to models with any number of DOF or having higher structural complexity.

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